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A MODEL FOR OPTIMIZING FIELD ARTILLERY FIRE

by

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A Model for Optimizing Field Artillery Fire

by

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Captain, United States Army
B.S., United States Military Academy, 1979

Submitted in partial fulfillment of the
requirements for the degree of

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March 1989

ABSTRACT

A microcomputer-based optimization model for short-term allocation of field artillery fire is developed and evaluated. The Artillery Optimization Model utilizes a mixed integer linear program that takes available targets, weights the targets by performing Target Value Analysis, and assigns firing units specific amounts and types of ammunition to fire at designated targets. In determining the optimal near-term allocation of artillery resources the model considers the target's intrinsic value, current ammunition levels, future ammunition re-supply, capabilities and limitations of the firing units, the ability of the artillery to mass fires, and the commander's criteria for target destruction. The model has been evaluated via direct competition with three experienced artillery officers using the Janus(T) high-resolution combat simulation. The results of the evaluation have shown that the Artillery Optimization Model produces a greater destruction, per projectile, than any of the artillery officers. If the results of the evaluation are projected over the course of a battle, the combat power of the field artillery would be substantially increased using the Artillery Optimization Model.

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I. INTRODUCTION

A. GENERAL

The field artillery is known as the "King Of Battle" because of its ability to inflict massive damage on enemy forces. However, resources such as the available artillery units, ammunition, and time dictate that the field artillery cannot engage all available targets on the battlefield. The field artillery fire support system must decide which targets warrant field artillery fire, and what is the best manner with which to attack those targets.

Given that the next battlefield can be described as "target rich", the possible combinations of targets, ammunition and weapon systems necessitate the use of an automated target processing system. The current methodology employed by the TACFIRE system normally attacks targets on a first in, first out basis [Ref. 1: p. 6-15], with no regard for future ammunition levels [Ref. 1: p. 6-9]. Thus, TACFIRE does not maximize the potential of the field artillery. In fact, one of the requirements of the *Organizational and Operational Plan* for the future command and control system of the artillery, known as the Advanced Field Artillery Tactical Data System (AFATDS), is that

> AFATDS will develop specific instructions for target attack. It will determine the method of engagement (projectile fuze combination and number of rounds the weapons are to fire). [Ref. 2: p. 8]

Thus, the Army recognizes the need for an advanced fire control system that manages field artillery fires.

This thesis will present a model called the Artillery Optimization Model. The purpose of the Artillery Optimization Model is to quickly prioritize targets and then engage selected targets using an optimal allocation of field artillery assets.

B. CHARACTERISTICS OF THE ARTILLERY OPTIMIZATION MODEL

The Artillery Optimization Model is microcomputer-based and utilizes a mixed integer linear program that takes available targets, weights the targets, and assigns firing units to the targets based on the following criteria:

1. The target's intrinsic value;
2. The characteristics of the artillery systems;
3. The capabilities and limitations of the firing units;

4. Available ammunition;
5. Expected ammunition resupply;
6. Commander's guidance;
7. The ability of the artillery to mass fires;
8. The fact that each round fired by an artillery unit increases the probability of detection by enemy forces.

The Artillery Optimization Model is a proof prototype model for a real-time decision support system for optimizing field artillery fire.

C. DEFINITIONS

The following definitions apply in this thesis.

1. A round is a synonym for projectile.
2. A volley is a unit firing a certain type of ammunition, in unison, at the same target. For example, if each howitzer in a unit fires 4-rounds of high explosive ammunition at a particular target, this is the same as 4-volleys of high explosive.
3. An element refers to an individual entity of a particular artillery unit. For example, the elements of a platoon are the howitzers while the elements of a battery are the platoons.
4. A mission is a gun or group of guns firing some number of successive volleys at the same target using the same type of ammunition. For example, a unit firing two volleys of high explosive ammunition at a target is firing one mission.
5. Adjust fire is the process of moving the impact location of the round, with one gun firing one round at a time, until the desired location is achieved.
6. Fire for effect means that one or more howitzers fire a predetermined number of rounds at the target.
7. Massing artillery fires means simultaneously attacking the same target with several elements.

D. ORGANIZATION OF THE THESIS

Chapter 2 contains background information on the organization and employment of the field artillery. Readers familiar with current doctrine regarding the field artillery may only wish to scan these sections. Chapter 2 also contains a literature review. Relevant assumptions concerning the effects and employment of the field artillery that pertain to the Artillery Optimization Model are in Chapter 3.

The thrust of this thesis is Chapter 4, where the Artillery Optimization Model is fully developed and described. While variables and equations are presented in detail, sections have also been devoted to programming and calibrating the model.

Finally, Chapter V describes a test conducted using the Artillery Optimization Model with the Janus(T) high resolution, combat model. Outcomes from this test are analyzed in the appendices.

II. BACKGROUND

A. MISSION

Field Manual 6-20 states that the mission of the field artillery is to

> ...destroy, neutralize, or suppress the enemy by cannon, rocket, and missile fire and to assist in integrating all fire support into combined arms operations. [Ref. 3: p. 3-21]

Essential to the accomplishment of this mission is the organization of the different components of the field artillery. Although field artillery units are specifically tailored for different missions, there are basic elements that are relevant to every field artillery unit.

B. ORGANIZATION OF THE FIELD ARTILLERY

The relationship between different echelons of the artillery is dependent upon the type of unit. What follows is a listing of some essential sections for a generic, split battery, 155mm, self-propelled field artillery battery organic to a division artillery:

1 Battery Headquarters,

1 Battery Fire Direction Center (FDC),

8 Howitzers, and

Associated Service Support.

In a division artillery, the echelon above battery is battalion. The composition of a normal field artillery battalion includes:

1 Battalion Headquarters,

1 Brigade Fire Support Element (FSE),

3 Company Fire Support Teams (FIST),

1 Battalion FDC,

3 Howitzer Batteries, and

Associated Service Support.

Although the FSE and FISTs are organic to the artillery, they are usually associated with a maneuver (Infantry or Armor) unit. In a combat environment, the FSE and FIST collocate and work with their respective maneuver counterpart.

The level above battalion is division artillery (DIVARTY). A DIVARTY ordinarily consists of the following:

1 Division Fire Support Element,

1 DIVARTY Headquarters,

1 Division FDC,

4 Howitzer Battalions,

1 Target Acquisition Battery (Location of radar units), and

Associated Service Support.

The echelon above DIVARTY is corps, and above corps is Army.

C. THE FIRE SUPPORT GUNNERY TEAM

In order to accomplish its mission, the field artillery relies on the fire support gunnery team. The team consists of an observer, the fire direction center, and the firing unit.

1. The Observer

The observer serves as the eyes of the fire direction team [Ref. 4: p. 1-1]. Although the observer may be a soldier with binoculars or a sophisticated radar system, the responsibilities of the observer stated in *Field Manual 6-30* include detecting and locating suitable indirect fire targets [Ref. 4: p. 2-2].

A Fire Support Team (FIST) is one type of observer. While the FIST is a component of the field artillery, the FIST is usually associated with a maneuver unit. A FIST consists of a headquarters (minimum of four men) and forward observers. The FIST is responsible for managing fire support for the supported company's battle plans. Although the responsibilities of the FIST are numerous, one of the principal duties of the FIST is requesting and adjusting indirect fires. [Ref. 4: p. 2-2]

Another type of observer is a radar section. A radar section is a separate element from a FIST and does not usually share the same relationship with a maneuver unit. The principal duty of radar is to detect enemy artillery, mortar and rocket units.

2. The Fire Direction Center

The fire direction center (FDC) serves as the brains of the fire support gunnery team [Ref. 4: p. 1-1]. The FDC receives the request for fires from an observer and converts it to firing data and then to fire commands for the howitzers.

It is the FDC that determines the number of rounds needed to accomplish a mission and the appropriate shell fuze combination. In fact, *Field Manual 6-40* states

that "The most important step in performing a target analysis is determining the number and type of rounds required to produce a desired effect." [Ref. 5: p. H-6] A guide for choosing the number and type of rounds exists in the Joint Munitions Effectiveness Manuals (JMEM); however,

> Using JMEMs to determine attack data requires considerable time. Because of time constraints, use of JMEMs at battalion and battery FDC levels is not recommended for engaging targets of opportunity. [Ref. 5: p. H-6]

Therefore, a Fire Direction Officer (FDO) must rely upon his training and experience to choose the number of rounds and appropriate shell fuze combination to engage each target.

3. The Firing Unit

The firing unit acts as the brawn of the gunnery team [Ref. 4: p. 1-1]. It is at the firing unit level that the fuze is mated to the projectile and loaded into the howitzer along with the appropriate propellant charge. While the FDC computes the firing data, the howitzer crew "sets this data off" on the weapon and fires the round.

D. FIRE SUPPORT SYSTEM FLOW OF EVENTS

Although there are numerous variations to the basic artillery call for fire, a routine request for artillery support would consist of the following events. First, a target must be detected. Assuming the target is detected by a forward observer (FO), the target is then transmitted to a FIST Chief who ensures that the target is not a duplication of an existing target. The FIST Chief then takes appropriate action to have the target engaged.

A FIST Chief is usually associated with a particular battalion Fire Support Officer (FSO). The Battalion FSO decides if the target is worth engaging and makes a judgement as to the amount of artillery necessary to neutralize the target. If the battalion FSO believes that more artillery is required than is available at his level, he may request additional support from a brigade FSO. Likewise, a brigade FSO requests from division, and division requests from corps. FSO's, or their representatives at each level, make subjective evaluations of the targets and decide whether to pass them on to firing units or request additional support.

Once a target reaches a firing unit, an FDO determines if the target can actually be engaged by his particular unit. Assuming the firing unit is to engage the target, the FDO must decide on the actual amount and type of ammunition with which to engage the target.

Finally, the target location is converted into firing data and fire commands are sent to the howitzers where ammunition is loaded and the weapons fired.

Figure 1 depicts this flow of events.

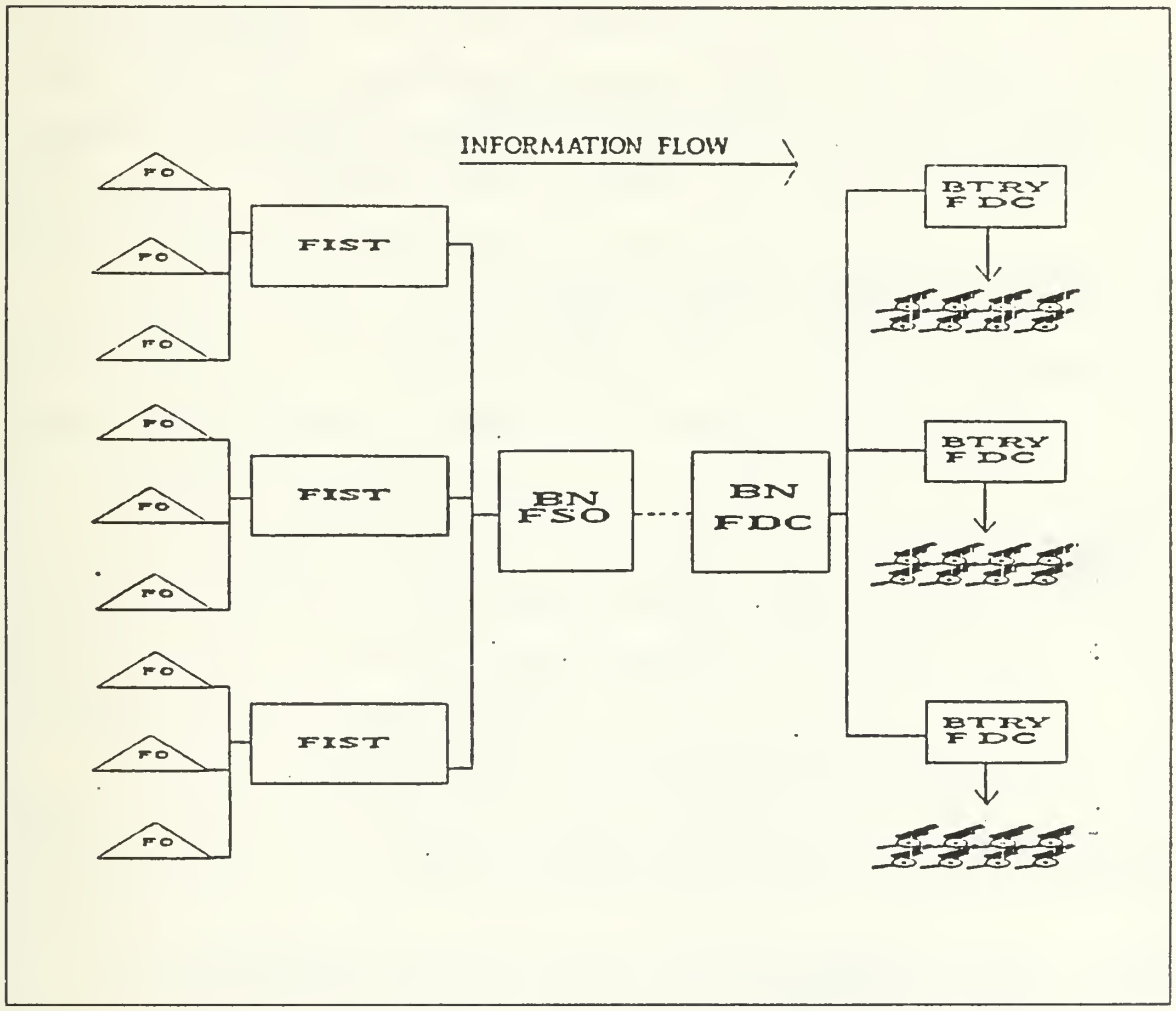

Figure 1. Fire Support System Flow of Events: A request for artillery fire usually originates with an FO and is processed through several channels until the firing data is computed by an FDC and transmitted to the howitzers.

E. LITERATURE REVIEW

Research into literature regarding decision support systems that allocate artillery fire led to the discovery of the following models.

1. Research Analysis Corporation Model

A Research Analysis Corporation (RAC) report entitled, *A Methodology for Determining Support Weapon System Mixes* [Ref. 6: 1973], develops a method "...for

determining the least cost mix of units which can accomplish the fire support tasks associated with a phase of combat." [Ref. 6: p. S-1] The key to the RAC methodology is the construction of a matrix in which the left column represents the fire support tasks and the top row represents candidate fire support units. The individual cells of the matrix are the number of units required to accomplish a specific task. An example matrix is depicted as Figure 2. Note that the RAC model does not limit the units to artillery; rather, units refer to any assets available, such as aircraft or howitzers.

Fire Support Tasks (λ_i)	Candidate Fire Support Units (μ_j)					
		μ_1	μ_2	μ_3	μ_4	μ_j
	λ_1	N_{11}	N_{12}	N_{13}	N_{14}	N_{1j}
	λ_2	N_{21}	N_{22}	N_{23}	N_{24}	N_{2j}
	λ_3	N_{31}	N_{32}	N_{33}	N_{34}	N_{3j}
	λ_4	N_{41}	N_{42}	N_{43}	N_{44}	N_{4j}
	λ_i	N_{i1}	N_{i2}	N_{i3}	N_{i4}	N_{ij}

Figure 2. RAC Fire Support Matrix: Entries N_{ij} represent the amount of unit j required to accomplish task i.

A linear program is used to assign the different tasks to the different units. A cost, b_{ij} is associated with each N_{ij}. The objective function is to minimize the cost of the fire support tasks. There are two types of constraints. The first constraint ensures that all i tasks are completed, and the second constraint ensures that enough of each unit is assigned to meet the need of that task.

Since the RAC approach utilizes a linear program, fractional units or weapon systems may be assigned tasks. Additionally, the solution generated by the RAC model may mix resources that are not operationally compatible. Further, all the advantages

or disadvantages of employing a certain resource are not accounted for in the cost. For example, "An F4 squadron may be compared to an artillery battalion for the role of fire support, but in doing so its air superiority role is ignored." [Ref. 6: p. 5-2]

2. Soviet Model

A Soviet report entitled *Automated Control Systems Provide Support to Artillery Fire* [Ref. 7: 1983], develops a method for distributing artillery resources. By implication, the first section of the report apparently considers the use of nuclear rounds by the artillery: this section only considers a single weapon firing a single round.

Section 3.3, *Rational Distribution of Artillery Fire*, concerns the "...distribution of enemy objectives among artillery battalions." [Ref. 7: p. 26] The model utilizes a matrix of the available units versus possible targets. The left column of the matrix represents artillery battalions and combinations of artillery battalions. The top row of the matrix consists of the targets.

Each target is preassigned a desired level of destruction that must be obtained if the target is to be engaged. Targets are classified as simple targets, which may be engaged with batteries from a single battalion, or complex targets, which may require more than one battalion to achieve the desired destruction. Additionally, targets are also classified according to importance groups.

The cells of the matrix consist of binary variables, designated as θ, that either allow or reject a given method of attack. Each cell also contains the number of batteries required to obtain at least the predetermined level of destruction, or in the case of complex targets, the portion of destruction obtained firing the entire battalion. Additionally, each cell contains the number of projectiles that would be expended.

A sample matrix is depicted at Figure 3. The required batteries are abbreviated as "Btry", battalion is abbreviated as "BN", rounds are abbreviated as "rnds" and T1 through T5 represent targets 1 through 5. Note that a dash indicates that the required level of destruction can not be obtained. For complex targets, the partial destruction coefficients are abbreviated by the term "Fill". One battalion is assigned the task of coordinating the fire support effort for the complex target, and that battalion is designated by not displaying a partial destruction coefficient.

Available Fire Support Units		Simple Targets			Complex Targets	
	Unit	T1	T2	T3	T4	T5
	Battalion 1	3 Btry 800 rnds θ_{11}	2 Btry 310 rnds θ_{12}	3 Btry 600 rnds θ_{13}	0.57 Fill 880 rnds θ_{14}	0.38 Fill 592 rnds θ_{15}
	Battalion 2	3 Btry 800 rnds θ_{21}	2 btry 310 rnds θ_{22}	2 Btry 320 rnds θ_{23}	0.66 Fill 898 rnds θ_{24}	0.44 Fill 598 rnds θ_{25}
	Battalion 3	3 Btry 1452 rnds θ_{31}	-	2 Btry 1200 rnds θ_{33}	2 Btry 1210 rnds θ_{34}	2 Btry 1210 rnds θ_{35}
	Battalion 1 and Battalion 2	-	-	-	1 Btry per Bn 310 rnds θ_{53}	-

Figure 3. Soviet Fire Support Matrix: Individual cell entries represent the number of batteries and amount of ammunition required to achieve a predetermined level of destruction. The binary variable θ_{ij} represents a particular method of attack.

An integer program is used to select which θ_{ij}'s appear in the solution. The primary objective function is to maximize the total θ_{ij}'s from the first importance group of targets. The solution is subject to constraints that allow the selection of only one θ_{ij} per target. Additional constraints limit the quantity of batteries employed to the number of available batteries, while ammunition is also limited to available ammunition.

If more than one optimal solution is calculated, secondary objective functions maximize the θ_{ij}'s in the second and third target importance groups. A final criteria minimizes the total expenditure of ammunition.

The Soviet model is not flexible in that a given level of destruction, per target, must be achieved even if a lesser ammunition expenditure would result in almost the

same level of destruction. Also, there is no method of distinguishing the effects of different ammunition. Finally, the model assumes the effects within a battalion are linear. For example, if only 50 percent of the effects from a battalion are required for a given target, then only 50 percent of the battalion need fire.

3. The Battle Decision Aid

A report entitled *Decision Support System for Fire Support Command and Control* [Ref. 8: 1983] describes a decision support aid developed for the United States Marine Corps. The name of this decision aid is *Battle*, and its purpose is to provide "...recommendations for the allocation of a set of weapons to a set of targets." [Ref. 8: p. 1]

The *Battle* decision aid has two phases. The first phase analyzes the effectiveness of weapons systems against the targets by "A complex calculation that uses 55 factors of the weapon, target and battlefield situation." [Ref. 8: p. 1] *Battle* uses a computation network to arrive at these effectiveness values.

The second phase computes a total amount of destruction based upon the effectiveness of the weapons targets calculated in phase one. The second phase uses these values, along with a tactical value for each target, to arrive at a solution that maximizes the total expected destruction.

Battle was tested using an eight weapon, seventeen target scenario. A Marine Corps artillery expert "...judged the allocation plans generated by battle against his expertise and found the plans to be acceptable solutions for the destruction of the targets." [Ref. 8: p. 11]

Limitations cited by the authors of reference 8 are that *Battle* "...delivers only one volley to target, does not schedule weapons fire, (and) does not assign munition fuse type." [Ref. 8: p. 19]

4. Literature Summary

The purpose of the Artillery Optimization Model is real-time target prioritization and fire mission assignment. The RAC model is designed to evaluate weapon system mixes, and treats weapon-to-target assignments in highly aggregate net assessment terms. The Soviet model is on a larger scale, disregarding ammunition expenditure in lieu of target destruction. *Battle* appears to be an intricate decision model rather than an effective decision aid.

The Artillery Optimization Model fills the need for specific, near-term decision support for optimizing field artillery fire.

III. ASSUMPTIONS

A. MISSION LIMITATION

The Artillery Optimization Model only considers missions dealing with target destruction. Special missions, such as illumination and smoke, are not considered by the model as these missions are concerned with target identification and obstruction, not target destruction.

B. METHOD OF FIRE

It is assumed that units will not adjust fire, rather, the observer's perceived target location is assumed accurate enough to allow a unit to fire in the "fire for effect" mode.

Chapter 6 discusses a method of accounting for targets that may require adjusting fire.

C. PROPELLANT CHARGES

The firing range of a projectile is a function of several items, including the choice of propellant charge. Since the desired range may be achieved using different propellant charges, it is assumed the propellant charge used is the one with the smallest expected range error.

D. AIMING AND BALLISTIC ERRORS

There are two types of errors which could cause a round to miss a target: the aiming error and ballistic error:

1. Aiming error, as depicted in Figure 4, is the difference between the desired aim point and the actual aim point.
2. Ballistic error, shown in Figure 4, is the error between the actual aim point and the location of the round's impact [Ref. 9: p. 2-28].

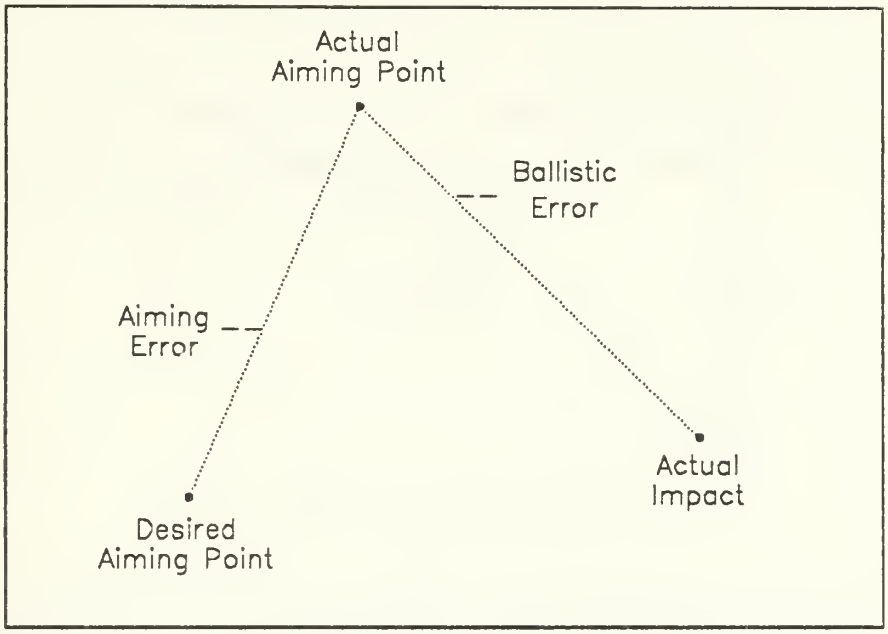

Figure 4. Ballistic and Aiming Error: Aiming error is the difference between the actual aiming point and the desired aiming point while ballistic error is the difference between the actual aiming point and the actual impact.

The Artillery Optimization Model accounts for aiming error by adjusting the value of the target. A target whose location is only estimated will be degraded in value whereas a target with an exact location is not degraded. Thus, the expected destruction tables, located in Appendix B, only allow for the ballistic error.

E. DESTRUCTION EFFECTS CURVES

The destructive power of the field artillery does not increase at a constant rate, rather, it increases at a decreasing rate. Plotting the expected destruction obtained from the Joint Munitions Effectiveness Manuals (JMEMs) for a given ammunition type against a specified target will yield a destruction curve as depicted in Figure 5. The Artillery Optimization Model assumes that the effects curve, within a volley, for a given target, unit and ammunition type is composed of linear segments. The approximate

shape of a destruction effects curve used in the Artillery Optimization Model is seen in Figure 6.

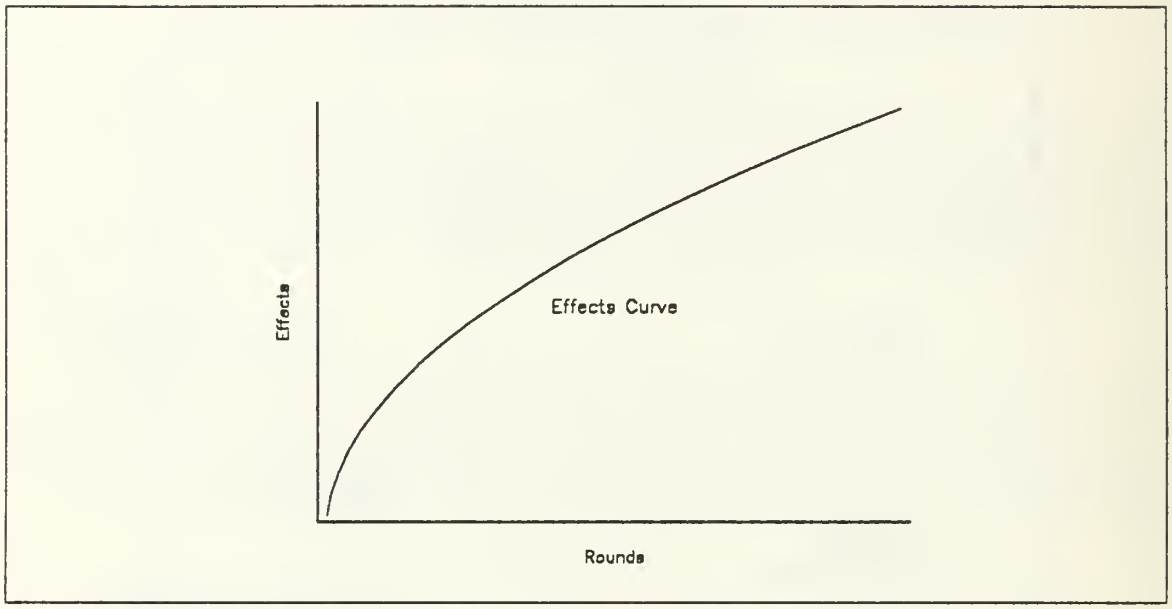

Figure 5. Destruction Effects Curve: The y-axis represents the amount of expected destruction for a specific target, while the x-axis represents the number of projectiles.

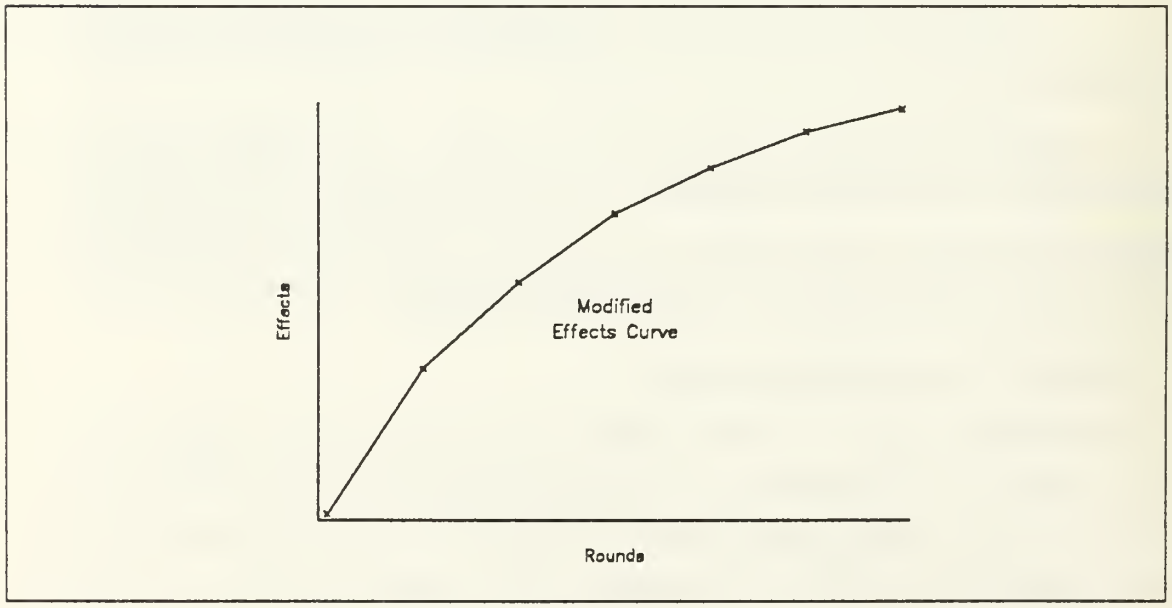

Figure 6. Modified Destruction Effects Curve: A piece-wise linear approximation is used to approximate the actual destruction effects curve.

The Artillery Optimization Model looks at each straight line segment of a destruction effects curve independently. Ammunition is defined by type as well as amount to be fired: for example, high-explosive 1-volley is a different type of ammunition than high-explosive 2-volley. The outcome is that an effects curve used in the Artillery Optimization Model closely approximates the actual artillery destruction curve for a given type of ammunition fired against a particular target.

IV. THE ARTILLERY OPTIMIZATION MODEL

A. PROBLEM STATEMENT

Field Manual 6-20 states one of the problems facing the field artillery in the accomplishment of its mission is that

> Weapons and ammunition are scarce, targets are plentiful, and the pace of battle is fast. [Ref. 3: p. 3-21]

Since field artillery assets on the battlefield are a limited resource, field artillery resources must be employed optimally. Two areas considered in optimizing field artillery fires are prioritizing enemy targets and optimally allocating artillery resources to inflict maximum damage on the enemy.

B. TARGET VALUE ANALYSIS

Although there is an abundance of targets on the battlefield, limited ammunition and artillery assets dictate that every available target cannot be engaged. A method is needed that allows the field artillery to quantitatively compare the importance of targets. Target value analysis (TVA) is a method of assigning numerical values to targets. Among the characteristics that TVA considers are [Ref. 10]:

1. Doctrinal value of a target;
2. Movement of a target;
3. Target mobility;
4. Target activity;
5. Situational weighting of a target;
6. Particular mission of the artillery unit.

TVA assigns a point value to each target, depending upon the target's attributes. One major criticism of TVA is that the doctrinal value of a target is, to some degree, a subjective judgement. Thus, commanders may differ with the doctrinal value based on their own experience and the current situation.

The Artillery Optimization Model allows for the difference in opinions concerning the value of a target by providing commanders the option of setting a desired destruction level for a particular class of targets.

C. THE MODEL

The Artillery Optimization Model utilizes a mixed integer linear program that optimally allocates artillery assets and ammunition to targets based on the TVA computed points. The following are the components of the Artillery Optimization Model.

1. Index Use

a. Target Number

The index used to represent targets is the letter i.

Potential artillery targets are designated with a sequential number that serves only to identify the target. Targets are then classified into target types.

b. Type of Ammunition

The index used to represent different types of ammunition is the letter j.

The possible amounts of each type of ammunition that can be fired at a target are considered as separate indices. For example, 1-volley of high-explosive ammunition is indexed differently from 2-volleys of high-explosive ammunition.

Additionally, the letter J represents each ammunition category, for example, $J = \{ \text{HE, ICM} \}$

c. Unit

The index used to represent units is the letter k.

A unit, for the purpose of this model, is defined as a group of firing systems acting in unison. While common units in the field artillery are the platoon, battery or battalion, the model does not restrict the term unit to those particular organizations. Any combination of artillery elements that should act in unison, must be designated as a unit. For example, in an artillery battalion that consists of three batteries, the following are conceivable units:

1. Each platoon.
2. Each battery.
3. Each possible combination of the three batteries.
4. The battalion.

2. Available Data

a. General Data

The following data is available under the guidelines used for the employment of the field artillery with the AFATDS system [Ref. 2].

(1) Target Quality Points. Q_i are the quality points for target i. Quality points are assigned to each target utilizing the concept of TVA. These quality points

are computed in a separate program and are assigned to each target before the model begins the optimizing process.

(2) Available Ammunition. $Ammo_{jk}$ is the available amount of type j ammunition for unit k. Although each j represents a different amount of ammunition, the ammunition available will be the same for all $j \in J$. The ammunition status is updated as unit k expends each category of ammunition.

(3) Expected Destruction. D_{ij} is the expected damage to target i given that a unit fires type j ammunition. Expressed as a percentage, values for D_{ij} are located in the Joint Munitions Effectiveness Manuals (JMEMs). These manuals "...provide guidance for determining the expected fraction of casualties to personnel targets or damage to material targets." [Ref. 11: p. 2-2] However, there is no assurance that a given number of volleys will produce the exact amount of destruction predicted by JMEMs, rather, the JMEMs acts a guide. Since the probability of hitting a target, and subsequently destroying it is a function of range, D_{ij} is also range dependent.

(4) Desired Effects on Target. E_i is the desired effects on the target. Expressed as a percentage, this number represents the commander's desired destruction of a particular class of target. For example, a commander might desire 5% effects for personnel targets.

(5) Percentage of Available Weapons. PH_k is the percentage of artillery elements capable of firing in unit k. This number acts as a force multiplier and assumes that the damage caused by a unit, per ammunition type, is a linear function of the number of elements available. For example, if a normal cannon battery consists of 8 howitzers, and one howitzer is unavailable, then the model assumes that each volley fired is (7/8) as effective as a complete 8 gun battery.

b. Model Specific Data

Although the following data is specific to the Artillery Optimization Model, it is derived from information currently available in the artillery fire support system.

(1) Projectiles per Volley. NP_{jk} is the number of projectiles fired by unit k. This amount is computed by multiplying the number of assigned weapons in unit k, by the percentage of elements available (PH_k), by the number of volleys. For example, if unit k is assigned 4 howitzers, and all the howitzers are firing 3-volleys, then NP_{jk} is 12.

(2) Model Time Period. Δt is the time period for which the Artillery Optimization Model computes target values and optimizes the artillery fire. For

example, Δt might reasonably be about 2 minutes. The model review process could be nearly continuous, but the horizon is fixed at Δt.

(3) Maximum Tasks. $TAMax_k$ is the maximum number of tasks that a unit may perform in any given Δt. A task is defined as an additional duty performed by a unit, such as shooting at different targets or shooting different types of ammunition. Depending upon the state of training, available personnel, equipment and other details, some units will be capable of performing more tasks than others in a given Δt.

(4) Minimum Acceptable Quality Points. *QMin* is the minimum amount of quality points that a target must exhibit to be considered suitable for artillery fire. *QMin* is supplied as a model parameter.

(5) Minimum Acceptable Destruction. *TDMin* is the minimum acceptable target destruction per projectile. The purpose of *TDMin* is to establish a lower bound for the amount of quality points to be earned for any projectile fired. *TDMin* is equal to *QMin* multiplied by the expected destruction D_{ij} for one round of type J ammunition.

c. Movement Loss Factor

A field artillery unit cannot conduct numerous missions from the same location and expect to survive on the modern battlefield. Every mission a unit shoots from the same position increases the probability of that unit being detected by enemy forces. When a unit moves it cannot maintain a firing capability, thus, for the period that the unit is in transit it cannot fire any missions. Since targets are not engaged during a movement, some amount of enemy destruction (quality points lost) is forfeited as a result of this movement. The Movement Loss Factor (MLF_k) penalizes the unit, before the unit moves, for the potential loss of quality points by imposing a penalty on each round fired based upon the unit's subsequent move.

The following data is used in conjunction with the Movement Loss Factor.

1. *XMax* is the maximum number of rounds that a unit will fire from a given location. This number is usually predetermined by a commander.
2. *MT* is the expected movement time for unit k from its current position to its next position. The following factors comprise *MT*.
   a. "March Order" time, the time required to prepare the unit for movement;
   b. The expected travel time from the current position to the next position; and
   c. Occupation time, which is the time required to emplace the unit in the new position and prepare the unit to fire.

3. $MPMin$ is the minimum expected quality points lost due to a unit's relocation. $MPMin$ is defined as:

$$MPMin = \frac{MT}{\Delta t} \times TDmin \times PH_k \quad . \tag{3.1}$$

Note that $MPMin$ assumes that each howitzer fires during each Δt . This assumption is based on the belief that the pace of the battle will be fast and targets will be plentiful. The model can be calibrated to allow for a slower paced, less intense battle. Calibration is discussed later in this chapter.

Assuming that $MPMin$ is the amount of quality points forfeited because a unit moves, then a unit pays this price after it fires $XMax$ rounds. Likewise, if a unit has not fired any rounds then it has not forfeited any quality points. Figure 7 assumes a linear relationship between the number of rounds fired and the prospective quality points lost.

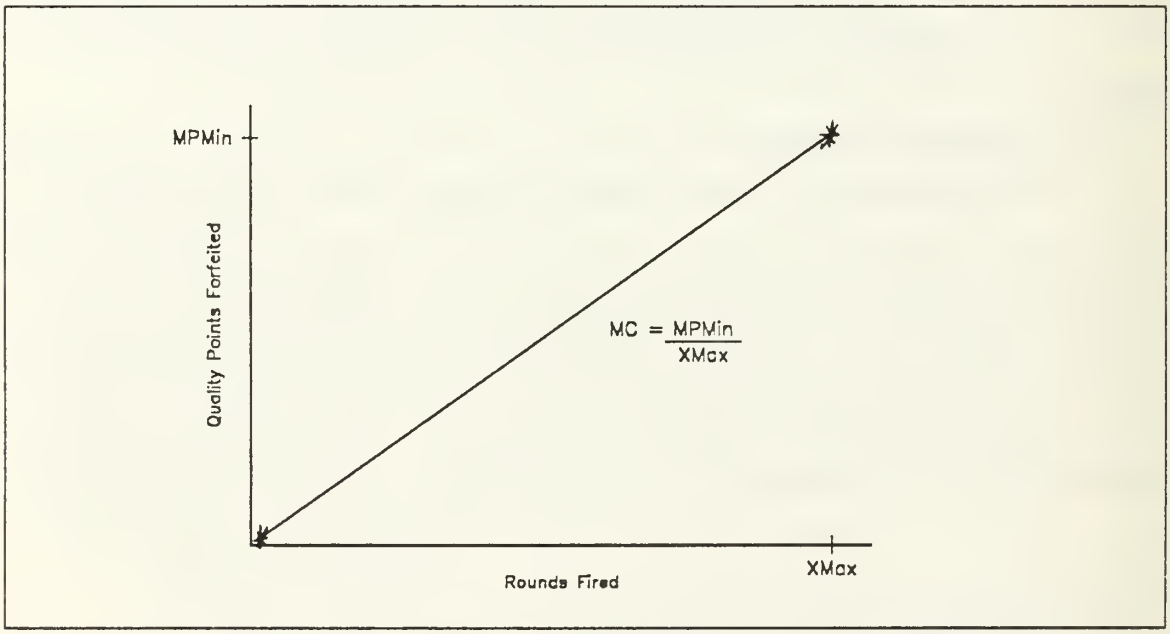

Figure 7. Quality Points Forfeited vs Rounds Fired: The quality points lost per each round fired is defined as the Movement Cost (MC) of firing each round.

The slope of the line in Figure 7 is the quality points lost per each round fired or simply the "movement cost" of firing each round. Thus, equation 3.2 defines movement cost (MC):

$$MC = \frac{MPMin}{XMax} \quad . \tag{3.2}$$

Finally, in order to penalize each round the appropriate movement cost, the total rounds fired for the current Δt are multiplied by the movement cost (MC). Equation 3.3 defines the MLF_k:

$$MLF_k = MC \times (\textit{Total Rounds Fired in Current } \Delta t) \quad . \tag{3.3}$$

Figure 8 shows the intent of the MLF_k. The destructive effects of artillery increase at a decreasing rate while the MLF_k is a linear increasing function. The intersection of the two curves is the point where the destruction obtained for a particular target is equal to the MLF_k. Firing any more ammunition would result in the MLF_k (penalty) being greater than the destruction (benefit), thus the MLF_k effectively acts as an upper-bounding influence.

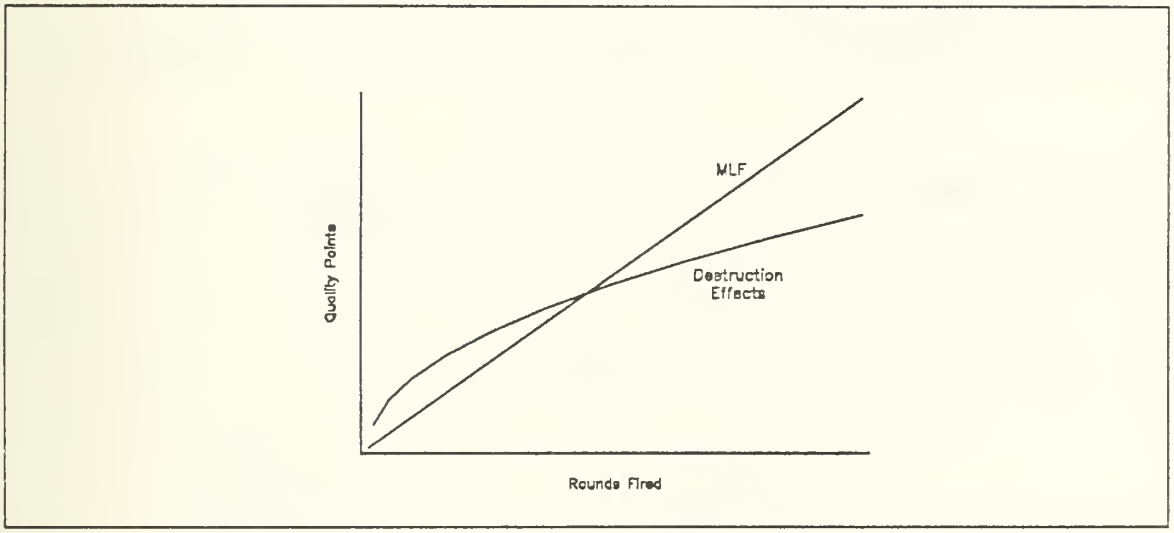

Figure 8. Intent of the MLF: The MLF is an upper-bounding influence that limits the number of rounds fired at a target.

d. Ammunition Loss Factor

Since field artillery units possess a limited amount of ammunition, and without ammunition units cannot accomplish their mission, *Field Manual 6-30* states that the Fire Direction Officer should "...select a weapon/ammunition combination that can achieve a desired effect with a minimum expenditure of ammunition stocks." (Ref.

5: p. II-4) The Ammunition Loss Factor (ALF) penalizes target ammunition combinations that result in excessive expenditures of ammunition.

An artillery unit will possess a certain amount of ammunition. This quantity is called ammunition that is "on hand". Of the ammunition on hand, a commander may dictate part of the supply be held in reserve until a crucial point in the battle. In any case, a commander usually knows how much ammunition his unit can expend until it is resupplied. $AMax_j$ is the maximum amount of ammunition that a unit may expend in a given time period. The time period in which a unit can expend $AMax_j$ is indicated as *Tammo*. Note that *Tammo* is generally much larger than Δt.

Since there is no way of predicting the mission-by-mission ammunition expenditure of a unit, Figure 9 assumes a linear relationship. The two known points are at time $t = 0$ when no ammunition has been expended, and at time $t = Tammo$ when a maximum of $AMax_j$ ammunition has been expended. The line connecting the two points is the "ammunition expenditure line" and represents a constant rate of ammunition expenditure for a given time period.

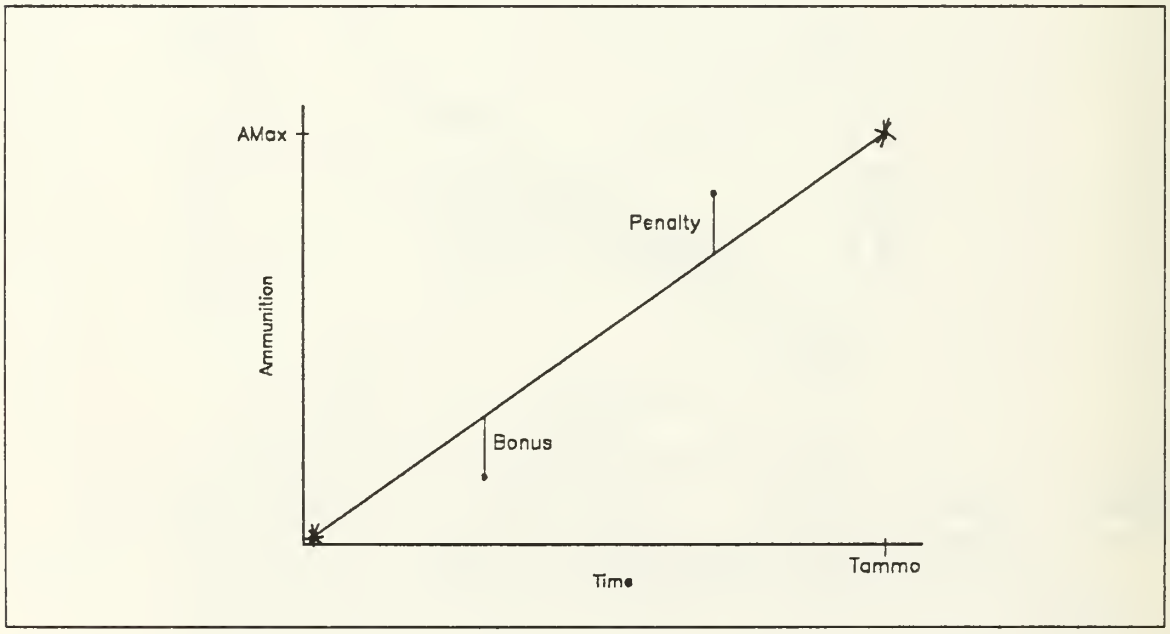

Figure 9. Ammunition Expenditure Line: Target Ammunition combinations below the ammunition expenditure line are given bonus points for conserving ammunition, while combinations above the line are penalized.

The intent of the $ALF_{j,k}$ is to penalize a unit for expending above the ammunition expenditure line, and to reward expenditures below the ammunition expenditure line.

The method used to compute the ALF_{jk} is as follows:

1. Determine $XOpt$, which is the desired ammunition expenditure (Y-axis) for a known time period (X-axis). Since the ammunition expenditure line is a straight line, the slope intercept formula gives the following equation:

$$XOpt = \left(\frac{AMax_j}{Tammo} \right) \times Desired\ Time\ Period \quad . \qquad (3.4)$$

2. Since the definition for $TDMin_j$ is the minimum acceptable target destruction, the ALF_{jk} penalizes a unit in $TDmin_j$ increments for expending above the ammunition expenditure line, and awards a unit in $TDMin_j$ increments for expending below the ammunition expenditure line.

The ALF_{jk} is computed based upon the number of rounds consumed above or below the ammunition expenditure line for the current time period. The following equation defines the ALF_{jk}, for each unit k.

$$ALF_{jk} = TDMin \times \left(XOpt - \begin{array}{c} Previous\ Type\ j \\ Ammunition\ Expended \end{array} - \begin{array}{c} Total\ Rounds\ Fired \\ in\ Current\ \Delta t \end{array} \right) \qquad (3.5)$$

Since the ALF_{jk} is computed by ammunition type, and by unit, the ALF_{jk} will effectively ensure that all units are expending comparable amounts of ammunition. Additionally, early in the battle the ALF_{jk} will discriminate against the lower quality point targets. If the pace of the battle is slow, and ammunition expenditure is low, then the ALF_{jk} will permit a more liberal target engagement policy until expenditure levels approach the ammunition expenditure line.

e. Commander's Desired Effects

Since an actual combat situation might cause the commander conducting the battle to weight targets differently than the doctrinal value, a method is needed that allows the model to account for the desires of the commander.

The Artillery Optimization Model will specifiy an acceptable range of δ_i percent within which the commander's desired effects are considered effectively satisfied. Damage less than δ_i percent below the desired effects will be penalized by parameter $\underline{\phi}_i$, while damage greater than δ_i percent above the commander's desired effects will be penalized by $\overline{\phi}_i$.

3. Model Variables

a. Decision Variables

$X_{i,j,k}$ is a binary decision variable representing an option to attack target i, using type j ammunition, fired by unit k. Since $X_{i,j,k}$ is binary, then either this method of attack is utilized or rejected.

b. Artificial, Slack, and Surplus Variables

The following variables are created to adjust the output of the model with respect to the commander's criteria for target destruction.

(1) Artificial Variable. A_i is an artificial variable that accounts for a method of attack that does not meet the commander's desired effects. The range of A_i is from 0 to infinity. The coefficient $\underline{\phi}_i$ weights the importance of not meeting the commander's criteria for this target.

(2) Surplus Variable. R_i is a surplus variable that accounts for a method of attack that is greater than the commander's desired effects. The range of R_i is also from zero to infinity. The coefficient $\overline{\phi}_i$ weights the importance of surpassing the commander's criteria.

(3) Slack Variable. S_i is a slack variable that allows a method of attack to be within a certain range of the commander's desired effects before the method is penalized. This range is determined by δ_i which is the percent error allowed. In other words, the actual effects may differ by $\pm \delta_i$ percent before a penalty is assessed.

The application of these variables allows the Artillery Optimization Model to consider the commander's desires even if their strict enforcement would preclude an effective method of target engagement.

4. Objective Function and Constraints

The objective function is to maximize quality point destruction, subject to the movement loss factor and the ammunition loss factor. Additionally, a penalty is imposed to account for a solution that either fails to attain or exceeds the commander's criteria for target destruction.

$$\text{MAX} \sum_i \sum_j \sum_k \Big[(Q_i D_{ij} PH_k) - (MC)(NP_{jk}) \tag{3.6}$$

$$+ TDMin\,(\,XOpt - \textit{Previous Type J Ammunition Fired} - NP_{jk}\,)\Big] X_{ijk}$$

$$- \left(\sum_i \underline{\phi}_i A_i + \overline{\phi}_i R_i \right)$$

Subject to the following constraints:

$$\sum_j \sum_k \left(Q_i D_{ij} PH_k \right) X_{ijk} + A_i + S_i - R_i = E_i Q_i \qquad \text{ALL } i \tag{3.7}$$

Where $0 \leq S_i \leq 2\delta_i E_i Q_i$

$$\sum_i \left(NP_{jk} PH_k \right) X_{ijk} \leq Ammo_{jk} \qquad \text{ALL } jk \tag{3.8}$$

$$\sum_i \sum_j X_{ijk} \leq TAMax_k \qquad \text{ALL } k \tag{3.9}$$

$$\sum_{j \in J} \sum_k X_{ijk} \leq 1 \quad (\text{J represents each ammunition category}) \qquad \text{ALL } iJ \tag{3.10}$$

Equations 3.7 account for the difference between the commander's desired effects and the expected effects. The artificial variable, A_i, and the surplus variable, R_i, are repeated in the objective function with coefficients $\underline{\phi}_i$ and $\overline{\phi}_i$ which weight the respective variables. The range of the variable S_i allows for methods of attack that are within $\pm\,\delta_i$ percent of the commander's desired effects.

Equations 3.8 limit the ammunition expended by a unit. Usually, $Ammo_{jk}$ will be the ammunition on hand or an amount prescribed by the commander.

Equations 3.9 allow the commander to restrict the number of tasks that each unit may perform in any given Δt.

Equations 3.10 allow only one "unit" to attack a target, per ammunition type. Note that the term unit may be a combination of one or more different elements (i.e. $Unit_k$ = Battery A and Battery B). Additionally, equations 3.10 force the linear program to choose only one line segment on the destruction effects curve for each category of ammunition.

Equations 3.10 allow a target to be attacked by more than one category of ammunition. For example, a target may be attacked by both high-explosive and improved conventional munitions projectiles. This assumes that the effects of different categories of ammunition can be added together to produce the total effect. While this may not be accurate, no single method of determining the exact effects of mixed ammunition was available. The tactical importance of attacking a target with mixed ammunition, in the author's opinion, outweighs the error in assuming the effects can be added. However, the model may be restricted to allocating one category of ammunition per target by summing over all of j, not for $j \in J$.

5. Derivation of the Objective Function

Equation 3.6, the objective function, is presented in its final form. Originally, the ALF_{jk} and MLF_k existed as separate functions in the objective function. The following equation presents the objective function in its original form:

$$\text{MAX} \sum_i \sum_j \sum_k (Q_i D_{ij} PH_k) X_{ijk} + ALF_{jk} - MLF_k \qquad (3.11)$$

$$- \left(\sum_i \underline{\phi}_i A_i + \overline{\phi}_i R_i \right)$$

Equations 3.3 and 3.5 show both the ALF_{jk} and MLF_k contain the term "current rounds fired in this Δt"; this term is actually the decision variable, X_{ijk}, multiplied by the quantity $(NP_{jk} PH_k)$. Replacing the ALF_{jk} and MLF_k by equations 3.3 and 3.5., and applying algebra leads to the objective function presented in equation 3.6.

D. PROGRAMMING THE ARTILLERY OPTIMIZATION MODEL

1. Programming Components

There are three basic programming components to the Artillery Optimization Model.

1. Generating the problem;
2. Computing the optimal solution; and
3. Generating a report.

The Artillery Optimization Model was programmed on an IBM compatible 286 PC-AT using a problem generator and report generator written by the author and a optimizer for personal computers named MILP88, which is a product of Eastern Software Products, Inc..

Commander Mike Olson introduced the author to MILP88 [Ref. 12: p. 25]. Since MILP88 is compatible with several formats of input data, the author decided to write the problem generator and the report generator in the programming language with which he felt most comfortable, *Turbo Basic*, a product of the Borland Corporation.

2. Problem Generator

The problem generator consists of two separate programs and uses three data files that contain the target data, ammunition data, and the commander's desired effects.

The first program in the problem generator, MAKECOEF (short for make coefficients), creates the objective coefficients using the data in the target file and ammunition file. The information in the target file consists of the target type, the distance from the firing batteries and status of the target (stationary or moving), while the ammunition file contains the ammunition status for each unit and the current time period.

The second program, FORMAT, compares the objective coefficients to a predetermined acceptance level and formats the data for use by the optimizer.

MAKECOEF is approximately 200 lines of code while FORMAT is almost 500 lines. The combined running time for these programs is usually under 3 seconds, with the majority of that time spent reading from and writing to data files.

3. Report Generator

The report generator consists of a single program, named REPORT, of approximately 160 lines of code. The report generator uses the solution generated by MILP88 as input, and translates the solution into a fire mission which consists of a target, artillery unit, amount of ammunition, and ammunition category. Additionally, REPORT updates ammunition expenditures in the ammunition file.

4. Reduction Of Variables

Techniques are available that may reduce the number of variables considered by the optimizer. Reducing the number of variables will make the model easier for the

optimizer to solve given the limited amount of time available. These techniques are described below.

a. Pre-Screening Variables

Some target ammunition or target battery combinations will be tactically or logistically infeasible. For example, a negative variable coefficient implies a target ammunition combination that produces little expected destruction in consideration of the ammunition expended. Such combinations could be eliminated with virtually no effect on the optimized solution.

These coefficients can be compared to a predetermined acceptance level with coefficients less than the acceptance level being eliminated. Note that the acceptance level may differ according to the target type, ammunition type or battery.

A second method of pre-screening variables considers the composition of the objective coefficient of each decision variable. The following equation relates the three components of the decision variable coefficient:

$$\text{Objective Coefficient} = \frac{\text{Destruction Effects}}{\text{Curve Value}} + ALF_{jk} - MLF_k \qquad (3.12)$$

The destruction effects curve increases at a decreasing rate while the MLF_j and ALF_{jk} are both linear functions. Additionally, the ALF_{jk} will reach a point where it becomes increasingly negative. Thus, once the objective coefficients of a sequence of decision variables begin to deteriorate, deterioration will continue for more rounds, and further terms need not be computed for that target battery ammunition category.

b. Selecting An Initial Incumbent

The method utilized by MILP88 to solve integer programs is the branch and bound procedure (e.g., see Garfinkel and Nemhauser, Ref. 13). Successive restrictions which develop enumeration branches are expensive to solve. There is a program control available in MILP88 which allows the user to specify an initial value for the incumbent, which may decrease the time needed to solve the problem [Ref. 14: p. 67]. The structure of the Artillery Optimization Model allows this technique to be exploited.

It is possible to show that if there is at least one positive objective coefficient in the Artillery Optimization Model, then there will always exist at least one solution to the model. Furthermore, the minimum value for the objective function can be easily computed and used as the initial incumbent. Thus, it will be assumed there exists

at least one positive objective coefficient and it will be shown that a single decision variable does not violate any constraints.

Since equation 3.7 is an equality constraint with slack and surplus variables, equation 3.7 will not effect the feasibility of the model.

Equation 3.8 is the ammunition constraint. It is logical to assume that there is sufficient ammunition to satisfy any individual decision variable considered by the optimizer. A decision variable that represents insufficient ammunition would be deleted in the pre-screening process.

Any single decision variable represents one task. Equation 3.9 allows a unit to perform a maximum of $TMax_k$ tasks. It is logical to assume that $TMax_k$ is greater than zero or else a unit could not perform any tasks and should be eliminated from consideration.

Finally, equation 3.10 allows one decision variable, per target and ammunition category, to be selected. A single decision variable does not violate this constraint.

Thus, a single decision variable with a positive objective coefficient does not violate any constraints. A solution for a single decision variable may be computed during the problem generation phase using equations 3.6 and 3.7. The maximum value obtained from a single decision variable is recorded and used as the initial incumbent.

Using the procedure outlined above for a 155mm howitzer battalion firing a maximum of 4-volleys of high explosive or improved conventional munitions, the time required to solve a seven target problem was reduced from over 16 minutes to under 25 seconds.

E. CALIBRATION

Calibration can be defined as fine tuning a model so that the output is consistent with accepted results or other approved solutions. Calibration is a powerful tool that may cause a significant difference in the model's results. There are three control parameters in the Artillery Optimization Model that may be adjusted in order to control the model's output. These control parameters are:

1. $TDMin$;
2. The objective coefficient acceptance level; and,
3. The commanders effects coefficients ($\underline{\phi}_i$, $\overline{\phi}_i$) and the range of the slack variables, S_i.

The methodology used in adjusting these parameters and the expected effects on the outcome of the model are discussed below.

1. **Adjusting TDMin**

The purpose of *TDMin* is to provide a basis for the minimum amount of acceptable target destruction. This minimum amount of acceptable destruction is utilized as a constant in both the MLF_k and the ALF_{jk} to penalize or reward different methods of attack.

TDMin can be changed by increasing or decreasing *QPMin* . Increasing *TDMin* will increase the value of the MLF_k, thus, a greater penalty will be assessed each round fired by a unit. It follows that as the slope of the MLF_k increases, the intersection of MLF_k and destruction effects curve will decrease, thus, the maximum allowable rounds fired at a certain target will be reduced. This phenomenon is pictured in Figure 10. Likewise, a decrease in the MLF_k allows a corresponding increase in the maximum allowable rounds fired at the same target.

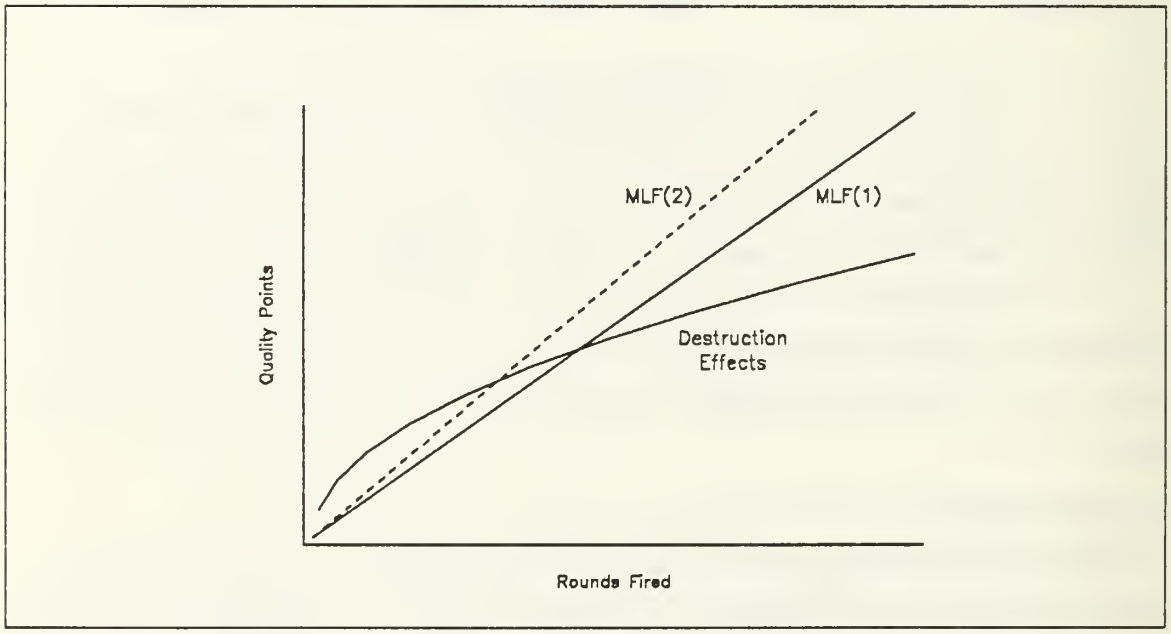

Figure 10. Effect of Increasing TDMin: Increasing TDMin changes the intersection of the destruction effects curve and the MLF_k, thus, decreasing the maximum rounds fired at a target.

Changing the value of *TDMin* also influences the ALF_{jk}. Since attack combinations are penalized or rewarded in *TDMin* increments, an increase in the value of *TDMin* will increase the penalty for high ammunition expenditures and increase the

reward for lower ammunition expenditures. Conversely, decreasing the *TDMin* will reduce the penalty and the reward.

2. Tuning the Objective Coefficient Acceptance Level

Decreasing the value of the acceptance level of the objective coefficients *reduces* the number of variables that are considered by the optimizer. Conversely, increasing the acceptance level will allow more variables to be considered by the optimizer.

The acceptance level may be used to ensure that only attack combinations meeting a specified level of expected destruction are considered by the model. However, an acceptance level that is too strict may eliminate feasible attack combinations, while an acceptance level that is too permissive would allow unrealistic attack combinations to be considered by the optimizer, possibly increasing the time to find a solution.

3. Adjusting the Desired Effects Coefficients

The artificial variable, A_i, accounts for a method of attack that does not meet the commander's desired effects. The objective coefficient of A_i is $\underline{\phi}_i$ and represents the weight applied to this criterion. A $\underline{\phi}_i$ equal to zero eliminates the influence of attaining the commander's desired effects while increasing $\underline{\phi}_i$ increases the influence of attaining the commander's effects to the point where it becomes a driving force for the entire model.

The surplus variable, R_i, accounts for a method of attack that exceeds the commander's desired effects. The objective coefficient of R_i is $\overline{\phi}_i$ and represents the weight applied to this criterion. A $\overline{\phi}_i$ equal to zero eliminates the influence of exceeding the commander's desired effects while increasing $\overline{\phi}_i$ increases the influence of exceeding the commander's effects to the point where it may also become a driving force for the entire model.

The range of the slack variable, S_i, is determined by the parameter δ_i, which is the acceptable error for not attaining or surpassing the the commander's desired target effects. Increasing the value of δ_i increases this range and lessens the chance that a penalty will be imposed. Likewise, decreasing δ_i increases the chance that a penalty will be imposed.

V. EVALUATION OF THE ARTILLERY OPTIMIZATION MODEL

A. GENERAL

The effectiveness of the Artillery Optimization Model was evaluated by comparing the methods currently used for artillery employment against the Artillery Optimization Model. The present system, called TACFIRE, processes targets and assigns artillery units to engage those targets on a first come, first serve basis [Ref. 1: p. 6-15]. At every level in the TACFIRE system, officers have the option to manually override the TACFIRE selection. Thus, it is an artillery officer who ultimately decides which targets to engage and the ammunition to be fired. Accordingly, the Artillery Optimization Model was placed in a direct contest with the expert judgement of three field artillery officers. The high resolution combat model, Janus(T), was used to provide a suitable combat scenario for the comparison of the model and the artillery officers.

B. THE JANUS(T) MODEL

The operating manual for the Janus(T) system describes the Janus(T) model as an interactive, two-sided, closed, stochastic, ground combat simulation. Interactive means that the players control, react and direct the operation of their assigned units. Two-sided implies there are opposing forces, and closed means that opposing players do not completely know the disposition of each other's forces. [Ref. 15: p. 6]

Janus(T) players plan and conduct tactical operations and make decisions by using interactive graphics work stations. Players make decisions based upon a continuous presentation of the battle on a map-like display and on-call status reports. [Ref. 15 : p. 6]

Since the Janus(T) simulation was used to create the same combat scenario for the officers and the Artillery Optimization Model, both the officers and the model could independently select which targets to engage and determine the appropriate unit ammunition combination.

C. TEST DESCRIPTION

1. Test Purpose

Several measures of effectiveness (MOEs) will compare the performance of the Artillery Optimization Model with the performance of the artillery officers, however, this

test will not provide conclusive evidence concerning the validity of the Artillery Optimization Model.

2. Test Design

Two different test scenarios were created using the Janus(T) model. Both scenarios utilized European terrain with a Red armor regiment attacking a Blue armor task force. Additionally, numerous targets, such as Ammunition Supply Points and Headquarters elements, were created and made known to each artillery player at the same time during the battle.

The same scenarios were used for all participants, including the Artillery Optimization Model. The only differences in the scenarios are the selection and method of engagement of targets by the artillery players. To ensure that the scenarios remained the same, the artillery destruction parameters on the Janus(T) model were set to zero.

Three experienced, U.S. Army field artillery officers were chosen as Blue artillery players. These officers were responsible for deciding which targets to attack, the type and quantity of ammunition to use, and which unit to fire.

The Blue artillery force consisted of one, 155mm self-propelled, howitzer battalion, divided into eight split-batteries of four guns per split battery. Since Janus(T) only allows complete units to fire, the Blue artillery players effectively had eight split batteries of 155mm under their control.

3. Blue Artillery Ammunition

Artillery players were limited to the use of high explosive (HE) and improved conventional munitions (ICM) projectiles. Table 1 shows the ammunition per Blue howitzer at the start of each scenario.

Additionally, players were instructed that an ammunition re-supply would not take place for two to four hours, thus, players were required to react to a realistic situation of limited ammunition.

Table 1. AVAILABLE AMMUNITION PER HOWITZER

Scenario No.	HE	ICM
I	45	30
II	30	45

4. Initial Round Time and Subsequent Round Time

Initial round time is the time required for the first round of a volley to be processed and fired. Subsequent round times are for rounds fired only during that same fire mission. These times follow in Table 2.

Table 2. INITIAL AND SUBSEQUENT ROUND TIMES

Weapon	Initial Round Time	Subsequent Round Time
155mm	45 sec	25 sec

5. Criterion

The coefficients $\underline{\phi}_i$ and $\overline{\phi}_i$ were both set to zero, thus no commander's criterion was established. Therefore, the target selection criterion for the individual artillery officers remained a product of their education and experience.

D. TARGET QUALITY POINTS

The list of target quality points is provided as Appendix A. Although this list does not provide quality points for every conceivable target, the Janus(T) simulation was structured so that all available targets could be classified into one of the listed categories. The quality points were obtained from an unclassified proposed target prioritization plan, reference 10, for the Advanced Field Artillery Tactical Data System (AFATDS).

E. EXPECTED DESTRUCTION VALUES

The expected destruction values located in the JMEMs were not used in this thesis due to their security classification. The object was to use an unclassified method of producing suitable expected destruction values.

Janus(T) uses a probabilistic mechanism to inflict damage from artillery fires. The Artillery Optimization Model needed to have some deterministic equivalent for the expected damage for each fire mission. Thus, the expected destruction values, located in Appendix B, were created using a "stand alone" artillery program for the Janus(T) model provided by the Training and Doctrine Analysis Command, White Sands, New Mexico.

The "stand alone" artillery program allows a user to conduct a fire mission by firing a predetermined amount of artillery against a group of targets while recording the effects of the artillery. Since the "stand alone" program bypasses unnecessary sub-programs, numerous replications of the same fire mission are possible in a relatively short period of time. By conducting numerous replications of the same fire mission, an average amount of destruction can be computed and utilized as the value for the actual amount of expected destruction.

The number of replications needed to ensure a level of significance of 0.2, ± 0.05, was calculated to be approximately 1050. Thus, 1050 trials of the "stand alone" artillery program were conducted for each target distance ammunition category listed in Appendix B.

A statistical test was not conducted comparing the derived destruction values to the JMEMs destruction values because of the security classification of the JMEMs values. However, the author employed his own judgement and artillery experience to determine if the derived destruction values were realistic. In situations where the derived value appeared to be erroneous, the author modified the value to reflect what he felt to be a more accurate value.

F. MASSING ARTILLERY FIRES

Massing artillery fires means simultaneously attacking the same target with several elements. An example of massing fires is firing one battery volley (8 guns), as opposed to firing two platoon volleys (4 guns). Although both instances fire 8 total projectiles, firing two volleys from one platoon allows the enemy to react after the first volley is fired and seek protective cover from subsequent volleys.

The destruction tables used in this thesis were created using the "stand alone" artillery program in conjunction with the Janus(T) simulation. The "stand alone" artillery program does not account for the effects of massing artillery fires, rather, damage is a function of the total number of rounds fired. To account for the concept of massing fires in the evaluation of the Artillery Optimization Model, a factor of 1.5 was used to multiply the effects of a platoon when platoons fired as batteries.

G. RESULTS

Some general observations concerning the artillery players are provided followed by more specific measures of effectiveness. Note that the term "player" refers to one of the artillery officers, "model" refers to the Artillery Optimization Model and "method" refers to the players and the model.

1. General Observations.

For all three artillery players, targets of opportunity became back-logged in a queue. Depending upon the player, the difference between the time the target was observed and the time the target was fired on was as great as 2 minutes. The artillery players commented that the problem of targets waiting in a queue was similar to their actual field artillery experience.

Since the information provided to the artillery players did not include a recommended method of attacking the targets, the artillery players were required to choose the amount and type of ammunition to fire at the targets of their choice. Since the players commented that it was somewhat awkward to change the number of volleys fired in the Janus(T) simulation, each player seemed to predetermine an optimal number of volleys that could be used to engage all types of targets. This number of volleys was rarely changed. The Artillery Optimization Model did not use a predetermined number of volleys to attack each target, thus, for targets of lesser value a smaller number of volleys was employed.

a. Quantity of Each Type Target Selected

A contingency table utilizing the Chi-square test for independence shows the Artillery Optimization Model, and the three artillery players, conducted approximately the same proportion of fire missions against each target category.

The results of the Chi-Square test indicate that for a level of significance of 0.05, the same proportion of each type target was selected regardless of the method employed.

The procedures used to construct the contingency table and perform the Chi-Square test are provided in Appendix C.

b. Percentage of Missions That Massed Fires

Field Manual 6-20 states that "A significant generator of immediate power is the ability of US fire support to mass fires." [Ref. 3: p. 3-1] Thus, the percentage of fire missions, for each method, that massed fires was observed and recorded.

Analysis of the data showed that the Artillery Optimization Model massed fires significantly more times than any of the artillery players.

2. Measures of Effectiveness

The following measures of effectiveness were used to compare the performance of the Artillery Optimization Model with that of the artillery players.

1. The expected quality points destroyed per round fired,

2. The total expected quality points destroyed per scenario, and
3. The expected quality points destroyed per fire mission.

a. Expected Quality Points Destroyed Per Round Fired.

The expected quality points destroyed, per type projectile, were computed by summing the total quality points destroyed and dividing this total by the number of rounds fired, for each type of projectile. This data is contained in Appendix E.

The data for both the HE and ICM projectiles showed that the Artillery Optimization Model displayed a greater expected points destroyed, per round, than any player. This difference is illustrated by the following example. The model average for the ICM projectile, for the combined scenarios, was 1.84 points per projectile. The closest player, player II, exhibited an average of 1.22 points per projectile. Thus, the model would be expected to destroy almost 51% more quality points, *per projectile*, than the closest player. If this difference were maintained throughout the course of a battle, the results would be significant.

b. The Total Expected Quality Points Destroyed Per Scenario

The total points destroyed is simply a measure of the total expected destruction by a particular method. These figures are contained in Appendix E. In all but one case, the Artillery Optimization Model destroyed more total points than any of the players.

In scenario II, for the ICM projectile, player II destroyed an expected 603 points where the Artillery Optimization Model destroyed an expected 531 points. However, player II expended 168 more ICM projectiles than the model. If the model maintained its 1.90 point destroyed per ICM round average that it exhibited in scenario II, and the model expended an additional 168 ICM rounds, then the model would be expected to destroy a total of 851 points.

c. Expected Points Destroyed Per Fire Mission

The expected points destroyed per fire mission, shown in Appendix E, gives an indication of how much destruction a target unit ammunition combination must exhibit to warrant artillery fire. For both scenarios, and both types of ammunition, the Artillery Optimization Model destroyed a greater amount of quality points per fire mission than any of the artillery players.

Additionally, the variance between the expected points destroyed, per fire mission, for the three players and the model were compared. The following hypothesis was tested:

$H_0 : \sigma_m^2 \leq \sigma_k^2$

$H_1 : \sigma_m^2 > \sigma_k^2$

where:

σ_k^2 is the variance for player k.

σ_m^2 is the variance for the model.

The test used to compare the variances was an F-test with a level of significance of 0.05. Equation 4.1 was used to calculate the test statistic, denoted as V, which was compared against an F-statistic value for $n_m - 1$ and $n_k - 1$ degrees of freedom, where n_m is the number of fire missions for the model and n_k is the number of fire missions for player k [Ref. 16: p. 622].

$$V = \frac{\sigma_m^2}{\sigma_k^2} \tag{4.1}$$

Table 3 contains the data for the HE projectile and Table 4 contains the data for the ICM projectile.

Table 3. COMPARISON OF VARIANCES FOR PROJECTILE HE

Method	σ^2	n	V	$F_{.05}$
Model	82.8	21	-	-
Player I	75.7	22	1.09	2.1
Player II	18.1	7	4.6	2.6
Player III	77.4	25	1.07	2.08

Table 4. COMPARISON OF VARIANCES FOR PROJECTILE ICM

Method	σ^2	n	V	$F_{.05}$
Model	182.2	40	-	-
Player I	190.4	30	0.96	1.7
Player II	196	56	0.93	1.5
Player III	148.8	29	1.22	1.79

The null hypothesis was accepted in every instance except one. For the HE projectile, player II, the null hypothesis would be rejected and the alternative hypothesis accepted.

Thus, for the expected points destroyed per fire mission, the model's variance was less than or equal to the variance for the players in all but one instance.

VI. CONCLUSION

A. SUMMARY

The Artillery Optimization Model utilizes a mixed integer linear program which optimally assigns target unit ammunition combinations that maximize the potential power of the field artillery. The model considers current ammunition levels, future ammunition re-supply, individual unit characteristics, commander's guidance and the ability of the artillery to mass fires in deciding optimal methods of attacking selected targets.

The model was evaluated using the Janus(T) high resolution combat model. Data from the evaluation suggests that since the three artillery players and the model selected essentially the same proportions of the different types of targets, differences in the expected destruction of targets must be a factor of how the targets were engaged. Since the Artillery Optimization Model produces a greater expected destruction per round than the players, the conclusion is that the model chooses a better combination of targets, ammunition and firing units.

Additionally, the Artillery Optimization Model masses fires significantly more than the artillery players. Thus, the model maximizes the potential power of the artillery to influence the battle.

B. MODEL STRENGTHS

The model is flexible in that it will try to achieve the commander's desired effects on targets. The model can be calibrated to weight the commander's guidance more or less than the doctrinal value of a target.

The Artillery Optimization Model is not restricted to one type of artillery unit. For example, the destructive effects of a 105mm battery may be compared to a 155mm battery using the appropriate JMEMs destruction values.

The model allows a commander to dictate how many tasks a unit is expected to accomplish in a given Δt. Thus, the model accounts for differences in unit strength, training and the current tactical situation.

A penalty is assessed if a unit begins to expend too much ammunition, thus the model will keep the percentage of ammunition expenditures relatively even for all units.

Since the model acts on all the targets received during a given Δt, the model may compare targets and strategy in computing an optimal allocation of artillery fire.

However, since targets are not attacked immediately, potential enemy targets are permitted to continue the battle until they are subsequently engaged.

A distinct advantage of the Artillery Optimization Model is that the model uses doctrinal criteria in assigning fire missions to units, thus, the ammunition expended and missions conducted are representative of what might be expected in actual combat. Therefore, the Artillery Optimization Model could realistically be used as an automated descriptive model to predict field artillery ammunition requirements, density of fire and the artillery's impact on the battle for a given combat scenario.

C. MODEL WEAKNESSES

Since the model operates myopically over a relatively short horizon of Δt, targets that require immediate attention may not be immediately engaged. However, this problem may be rectified by employing a continuous review version of the model with an horizon of Δt that causes the model to be employed whenever a target meeting certain specifications is observed.

The model assumes that processing fire missions requires a certain amount of time for each unit. If a unit constantly takes more time than expected, this will cause a backload of fire missions at that unit. This problem can be partially rectified by close supervision of the fire missions assigned to the units. If a backlog appears, the model may be instructed to eliminate a particular unit from consideration for future fire missions until the backlog of targets is fired.

The time required for a mixed integer linear program to find an optimal solution varies depending on the number of constraints and variables. For a battalion size artillery unit firing a maximum of four volleys of HE or ICM at eight different targets, there are over 800 variables and over 180 equations to be considered. While pre-screening may greatly reduce the number of variables, special programming will be required to ensure that the model produces an answer in a reasonable amount of time (2 - 3 seconds).

The optimizer used to solve the Artillery Optimization Model, MILP88, would usually solve problems with eight or fewer targets in under ten minutes, sometimes in under one minute. Larger amounts of targets usually exhausted the memory available on a personal computer and MILP88 was required to store parts of the problem on the hard disk, thus increasing the time to solve a problem by as much as several hours. However, this limitation is of no consequence to the adoption of the Artillery Optimization Model, since faster microprocessors are available at relatively modest costs.

For example, the Compaq 80386 with a Weitek 1167 coprocessor is claimed to work 16 times faster than the IBM 286 PC-AT and costs under $10,000 [Ref. 17: p. 22].

Ensuring that a solution is found in a relatively short period of time may necessitate the acceptance of an "almost" optimal solution. Hillier and Lieberman state that a nearly optimal solution can generally be found "...with much less computational effort." [Ref. 18: p. 412] The authors of the *Battle* command and control system discovered that the solution time decreased from approximately 12 minutes to about 7 seconds with the acceptance of a 98% optimal solution [Ref. 8: p. 1].

D. SUGGESTIONS FOR IMPROVEMENT

Currently, the model assumes each fire mission will be fired in the "fire for effect" mode. Since certain targets may require adjusting fire, the model should account for the extra ammunition and time required to adjust fire. The accomplishment of this task may require targets to be classified according to the manner in which they were observed.

Both the ALF_{jk} and MLF_j assume a linear relationship. The linear relationship may be replaced by a function that more closely approximates ammunition expenditure and probability of detection. Since these functions are not contained in the linear program, they need not be linear. The only requirement for the ALF_{jk} is that it be possible to calculate an optimal ammunition expenditure for a given time. For the MLF_j, the computation of penalty points must be possible for any possible number of rounds fired.

There may exist a target type that must be engaged, no matter what the consequences. In other words, if a certain target is observed, then it will be engaged. This problem may be handled by creating a pre-emptive constraint that forces the linear program to assign a firing unit to attack this target. However, the question arises, how much ammunition should be fired against this target? Assuming this target would not normally be selected, the model would then assign the smallest unit possible to engage the target with very limited ammunition. Artificially raising the value of a target might cause a concentration of artillery fire on a target that does not warrant it. Perhaps the best solution is to handle these types of targets by always firing a specified amount of ammunition whenever a particular type of target that must be engaged is observed.

APPENDIX A. TARGET QUALITY POINTS

The quality point values in Table 5 were obtained from reference 10.

Table 5. TARGET QUALITY POINTS

Target Name	Stationary	Moving
Medium Artillery	135	145
Heavy Artillery	151	146
Rocket Artillery	161	151
Platoon of BMP's	75	70
Company of BMP's	80	75
Battalion of BMP's	85	80
Company Assembly Area	88	-
Battalion Assembly Area	93	-
Regimental Assembly Area	98	-
Refuel Point or Ammo Supply Point	35	-
Truck Convoy	33	28
Bridge Equipment Company	23	18

Target Name	Stationary	Moving
Radar Site	80	75
Air Defense System	72	67
Company Headquarters	75	65
Battalion Headquarters	85	70
Regimental Headquarters	90	80
Bridges	40	-
Road Junctions	32	-
Infantry in Open	87	82
Attack Helicopters (On the Ground)	83	-
Tank Formations	88	78

APPENDIX B. EXPECTED DESTRUCTION TABLES FOR 155MM

The following table contains the expected destruction values for two types of 155mm ammunition. Each volley represents a platoon of four guns. Chapter 5 explains how these values were calculated. The distance for each calculation represents the *maximum*, inclusive distance for the given expected destruction value. Finally, a dash, (-), represents an expected destruction value less than .01.

Table 6. DESTRUCTION VALUES FOR 155MM HOWITZER

TARGET NAME	No. Volleys	High Explosive			ICM		
		6 km	12 km	18 km	6 km	12 km	18 km
Medium Artillery	1	.04	.03	.03	.13	.12	.09
	2	.07	.06	.05	.19	.17	.14
	3	.10	.09	.07	.24	.22	.19
	4	.12	.11	.09	.28	.24	.22
Heavy Artillery	1	.03	.03	.02	.11	.10	.09
	2	.05	.04	.03	.17	.16	.14
	3	.07	.05	.04	.20	.19	.18
	4	.08	.05	.05	.22	.21	.20
Rocket Artillery	1	.06	.04	.03	.15	.14	.12
	2	.10	.08	.08	.20	.19	.17
	3	.12	.10	.09	.25	.24	.22
	4	.14	.12	.10	.29	.26	.24
BMP Formation	1	-	-	-	.07	.06	.05
	2	.01	.01	-	.14	.12	.12
	3	.04	.04	.02	.18	.16	.15
	4	.05	.05	.03	.21	.19	.17
Assembly Area	1	.08	.08	.07	.15	.14	.11
	2	.13	.12	.10	.23	.23	.19
	3	.16	.15	.13	.30	.29	.27
	4	.17	.16	.15	.36	.34	.33

TARGET NAME	No. Volleys	High Explosive			ICM		
		6 km	12 km	18 km	6 km	12 km	18 km
Refuel Point or Ammo Supply Point	1	.09	.09	.08	.12	.11	.11
	2	.13	.13	.12	.17	.15	.14
	3	.17	.16	.13	.24	.22	.21
	4	.19	.18	.17	.27	.24	.22
Truck Convoy or Bridge Company	1	07	06	05	.19	.16	.12
	2	.16	.13	.12	.24	.22	.20
	3	.19	.16	.14	.27	.24	.22
	4	.19	.17	.15	.29	.25	.24
Radar Site	1	.21	.19	.12	.32	.30	.28
	2	.40	.37	.32	.50	.46	.43
	3	.51	.47	.41	.61	.58	.52
	4	.62	.55	.51	.68	.64	.58
Air Defense System	1	.09	.08	.05	.13	.12	.12
	2	.21	.18	.13	.25	.23	.21
	3	.28	.26	.21	.32	.28	.25
	4	.33	.30	.20	.37	.34	.32
Headquarters Elements	1	.07	.07	.06	.15	.14	.11
	2	.11	.10	.09	.26	.24	.23
	3	.14	.13	.11	.30	.27	.22
	4	.16	.15	.13	.33	.30	.25
Bridges and Road Junctions	1	-	-	-	01	-	-
	2	.01	-	-	.02	.01	-
	3	.02	.01	-	.03	.02	.01
	4	.02	.01	.01	.04	.02	.01
Dismounted Infantry	1	.09	.07	.06	.31	.28	.25
	2	.19	.16	.14	.49	.46	.43
	3	.24	.22	.21	.60	.57	.52
	4	.31	.25	.23	.65	.60	.55

TARGET NAME	No. Volleys	High Explosive			ICM		
		6 km	12 km	18 km	6 km	12 km	18 km
Attack Helicopters (On the Ground)	1	.05	.03	.03	.16	.12	.10
	2	.11	.8	.7	.20	.16	.15
	3	.14	.12	.09	.23	.19	.18
	4	.16	.14	.11	.25	.21	.19
Tanks	All	-	-	-	-	-	-

APPENDIX C. CONTINGENCY TABLE OF THE TARGETS SELECTED

A. HYPOTHESIS TESTED

In the evaluation of the Artillery Optimization Model, each of the three artillery players, and the model, were responsible for selecting which targets were to be engaged by artillery fire. Since the number of rounds expended on a target is a function of the desired destruction, the unit of measure chosen to count the targets selected was the number of fire missions conducted against a target. Thus, utilizing the number of fire missions per target, the following hypothesis was tested:

H_0 : The number of fire missions conducted against each target category is independent of the method used to select the targets.

H_1 : The number of fire missions conducted against each target category is dependent on the method used to select the targets.

The technique employed to test the above hypothesis was a contingency table utilizing a Chi-square test of independence.

B. THE CONTINGENCY TABLE

The Chi-square test compares the actual number of targets selected versus an expected number of targets selected. Duncan states that in order to properly conduct the Chi-square test, the conservative rule is to have an expected frequency of at least five observations per cell [Ref. 16: p. 52]. Thus, some of the target categories were logically combined to meet the five per cell criteria. The following list shows how the target categories were combined.

Headquarter Elements - all echelon headquarter elements.

Bridges and Road Junctions - all types of bridges and road junctions.

Enemy Artillery - all types of enemy artillery.

Maneuver Forces - all echelons of tank and mechanized infantry.

Soft Targets - Radar sections, Air Defense Artillery units, and Helicopters.

Logistical Units - Ammunition Supply Points (ASP), Re-fuel Points, and Bridge Companies.

Personnel Targets - Light Infantry, Assembly Areas.

Table 7 shows the actual number of each type of target selected by each method.

Table 7. ACTUAL NUMBER OF SELECTED TARGETS PER METHOD

Target Category	Model	Player I	Player II	Player III
Headquarters	7	6	5	5
Bridges and Road Junctions	0	7	9	6
Artillery	15	10	11	16
Maneuver Units	11	8	18	7
Soft Targets	16	8	9	8
Logistical Units	4	7	5	6
Personnel Targets	7	6	6	6

Using the Chi-square test for independence, the sample statistic is 20.75. Assuming a level of significance of 0.05, the Chi-square test statistic for 18 degrees of freedom is 28.9. Thus, the hypothesis that the number of fire missions conducted against each target category is independent of the method used to select the targets, is accepted.

APPENDIX D. PERCENTAGE OF MISSIONS THAT MASSED FIRES

The comparison of the proportion of fire missions that massed fires was tested using the Normal approximation to the Binomial distribution. Using the data from the combined scenarios, the following hypothesis was tested:

$H_0 : p_k \geq p_m$

$H_0 : p_k < p_m$

where:

p_k is the proportion of fire missions that player k massed fires.

p_m is the proportion of fire missions that the model massed fires.

Table 8 contains the appropriate data, along with the estimated values of $\hat{p}$, $\hat{\sigma}$ and test statistic, Z. The following procedure was employed to compute the values presented in Table 8 [Ref. 16: p. 606].

First, the Normal approximation to the Binomial distribution is good only if $pn \geq 5$ [Ref. 16: p. 100]. This condition was satisfied for all methods.

The following formula was used to obtain the values for $\hat{p}$:

$$\hat{p} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} \quad . \tag{D.1}$$

The values for $\hat{\sigma}$ were computed using formula D.2:

$$\hat{\sigma}_{p_1 - p_2} = \sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \quad . \tag{D.2}$$

Finally, the test statistic was computed using the following formula:

$$Z = \frac{p_1 - p_2}{\hat{\sigma}_{p_1 - p_2}} \quad . \tag{D.3}$$

Table 8. MASSING FIRES TEST RESULTS

Method	Number of Fire Missions (n)	Number of Times Massed Fires	p	$\hat{p}$	$\hat{\sigma}$	Test Statistic (Z)
Model	60	43	.717	-	-	-
Player I	52	11	.212	.482	.095	5.31
Player II	63	7	.111	.435	.089	6.81
Player III	54	12	.222	.482	.094	5.26

Comparing the test statistic with the value for a level of significance of 0.05 ($Z_{.05} = 1.645$), the test statistic is much larger in every case. Therefore, the null hypothesis is rejected for every player and the alternate hypothesis is accepted: the model massed fires significantly more times than the artillery players.

APPENDIX E. TEST RESULTS PROJECTILES HE AND ICM

The results from the evaluation of the Artillery Optimization Model were categorized by projectile, and follow as Table 9 and Table 10 of this appendix.

The combined totals for both scenarios were computed by combining all the fire missions for scenarios I and II into a single sample.

Table 9. SUMMARY STATISTICS FOR HE PROJECTILE

	Method	Total Fire Missions	Total Rounds Fired	Average Points Per Fire Mission	Standard Deviation Per Fire Mission	Average Points Per Round	Total Points Destroyed
Scenario I	Model	11	124	12.1	7.4	1.08	133.6
	Player I	12	192	7.2	6.6	.45	86
	Player II	7	112	2.7	4.2	.17	18.6
	Player III	16	256	12.0	8.6	.75	192.1
Scenario II	Model	10	148	16.7	10.5	1.13	167.1
	Player I	10	160	12.2	10.3	.76	121.8
	Player II	0	0	0	0	0	0
	Player III	9	144	12.2	10.3	.76	109.4
Combined Totals For Both Scenarios	Model	21	270	14.3	9.1	1.1	300.7
	Player I	22	352	9.5	8.7	.59	207.8
	Player II	7	112	2.7	4.2	.17	18.6
	Player III	25	400	12.1	8.8	.75	301.5

Table 10. SUMMARY STATISTICS FOR ICM PROJECTILE

	Method	Total Fire Missions	Total Rounds Fired	Average Points Per Fire Mission	Standard Deviation Per Fire Mission	Average Points Per Round	Total Points Destroyed
Scenario I	Model	18	268	26.5	15.7	1.78	476.3
	Player I	16	256	18.0	12.8	1.13	282.3
	Player II	28	448	17.5	12.4	1.13	288.3
	Player III	12	192	13.1	9.9	.82	156.9
Scenario II	Model	21	280	25.3	11.6	1.90	531.6
	Player I	14	224	18.9	14.8	1.18	264.7
	Player II	28	448	21.5	15.5	1.34	603.0
	Player III	17	272	19.0	13.1	1.19	323.7
Combined Totals For Both Scenarios	Model	40	548	25.9	13.5	1.84	1007.9
	Player I	30	480	18.4	13.8	1.15	553.0
	Player II	56	896	19.5	14.0	1.22	19.5
	Player III	29	464	16.6	12.2	.99	460.8

APPENDIX F. SAMPLE PROBLEMS AND SAMPLE RESULTS

A. GENERAL

The following tables provide results from the Artillery Optimization Model for the given target scenarios based upon artillery assets of one battalion of 155mm howitzers. The battalion consisted of three batteries, with each battery divided into two platoons of four howitzers. Each battery was allocated 360 rounds of high-explosive ammunition and 320 rounds of Improved Conventional Munitions (ICM). Re-supply would not take place for 4 hours.

The target quality points used are contained in Appendix A and the expected destruction values are contained in Appendix B. To account for the concept of massing fires, the destruction values were multiplied by 1.5 when platoons fired as batteries.

The solution time for both target scenarios was less than 25 seconds on an IBM compatible 286 PC-AT.

B. SAMPLE PROBLEM 1

Table 11 lists the available targets for the first sample result and Table 12 lists the firing solution computed by the Artillery Optimization Model. For this sample, the penalty for not meeting the commander's effects was four, and the penalty for surpassing the effects was one. The effects acceptance interval was $\pm$ 10%.

Table 11. TARGETS AVAILABLE FOR SAMPLE 1

Target Name	Quality Points	Desired Effects	Distance To Target (km)		
			Battery A	Battery B	Battery C
Heavy Artillery	151	.15	17	19	16
Truck Convoy	33	.1	17	18	22
BMP Company	85	.05	8	7	9
Attack Helicopters (on the Ground)	83	.12	15	17.5	16
Concrete Bridge	40	.005	6	5	6
Tank Company	88	.01	5	8.5	6
Air Defense System	72	.25	11	12	14

Table 12. RECOMMENDED FIRING DATA FOR SAMPLE 1

Target Name	Firing Unit	Number Volleys	Ammo Type	Expected Destruction	Desired Destruction
Heavy Artillery	1st Platoon Battery C	2	ICM	.14	.15
Helicopters	2nd Platoon Battery A	2	HE	.11	.12
Air Defense System	Battery B	2	HE	.27	.25

Only the targets with a higher expected quality point destruction were selected. Since the penalty for not meeting the commander's desired effects was relatively large, the expected effects for all three targets selected was within 10% of the desired effects.

C. SAMPLE PROBLEM 2

Table 13 lists the available targets for the second sample result and Table 14 lists the firing solution computed by the Artillery Optimization Model. For this sample, the penalty for not meeting the commander's effects was one, and the penalty for surpassing the effects was four. The effects acceptance interval was $\pm$ 10% .

Table 13. TARGETS AVAILABLE FOR SAMPLE 2

Target Name	Quality Points	Desired Effects	Distance To Target		
			Battery A	Battery B	Battery C
Medium Artillery	135	.17	19	14	16
Truck Convoy	33	.1	17	19	24
BMP Platoon	85	.05	9	6	7
Ammunition Supply Point	35	.22	15	12.5	15
Concrete Bridge	40	.005	6	5	6
Tank Company	88	.01	5	8.5	6
Radar Site	80	.35	13	12	15

Table 14. RECOMMENDED FIRING DATA FOR SAMPLE 2

Target Name	Firing Unit	Number Volleys	Ammo Type	Expected Destruction	Desired Destruction
Medium Artillery	Battery C	1	ICM	.135	.17
Ammunition Point	Battery B	2	HE	.195	.32
Radar Site	Battery B	1	HE	.18	.25

Again, only targets with a higher expected quality point destruction were engaged. Additionally, the fire missions for the selected targets were divided among the three batteries. Since the penalty for meeting the commander's desired effects was only one, and the penalty for surpassing the expected effects was four, the desired effects were never surpassed.

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